MATH 154 SPRING 2019-2020
Homework 1

Due date: February, 21, 2020

1. Let $f(x) = \begin{cases} 4\sin(\pi x) & -1 \le x < 1 \\ \sqrt{2x - x^2} & 1 \le x \le 2 \end{cases}$ on $[-1, 2]$

   (a) Compute the upper and lower Riemann sums of the function f over [-1,2] corresponding the partition P_3 of [-1,2] into 3 sub-intervals of equal length.

   (b) Compute the upper and lower Riemann sums of the function f over [-1,2] corresponding the partition P_6 of [-1,2] into 6 sub-intervals of equal length.

   (c) Calculate $\int_{-1}^{2} f(x)dx$ **geometrically**.(Do not use any integral formula)

   (d) Compare the results found above.

2. Consider the partition $P_n = \{x_0 = 1, x_1 = 2^{\frac{1}{n}}, x_2 = 2^{\frac{2}{n}}, ..., x_n = 2^{\frac{n}{n}} = 2\}$ of the interval $[1, 2]$. By using this partition, show that the area of the region bounded by $y = \dfrac{1}{x}$, $y = 0$, $x = 1$, $x = 2$ is $\ln 2$.

3. Express the area under the parabola $y = 4 - x^2$ and above $x - axis$ as the limit of a sum of areas of rectangles, and evaluate the area.

4. Let f be continuous and even on $\mathbb{R}$. If $\int_{0}^{153} f(x)dx = 7$, evaluate $\int_{-153}^{153} (11 + \sin x) f(x)dx$.

5. Express the following limits as definite integrals:

   a) $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \dfrac{5n}{n^2 + i^2}$

   b) $\lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \dfrac{153}{\sqrt{3in}}$

   c) $\lim\limits_{n\to\infty} \dfrac{4}{n} \sum\limits_{i=1}^{n} \sqrt{\dfrac{2(2i-1)}{n}}$

   d) $\lim\limits_{n\to\infty} \dfrac{1}{n}(\sqrt{3 - \dfrac{2}{n}} + \sqrt{3 - \dfrac{4}{n}} + \cdots + \sqrt{3 - \dfrac{2n}{n}})$

   e) $\lim\limits_{n\to\infty} S_n$ where

$$S_n = \frac{1}{n}\left(\frac{\ln(1 + \frac{1}{n})}{1 + \frac{1}{n}} + \frac{\ln(1 + \frac{2}{n})}{1 + \frac{2}{n}} + \cdots + \frac{\ln(1 + \frac{n-1}{n})}{1 + \frac{n-1}{n}} + \frac{\ln(2)}{2}\right)$$

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6**. (a) Prove that $\int_{ca}^{cb} f(t)dt = c\int_{a}^{b} f(ct)dt$.

   (b) By using (a), show that area enclosed by the ellipse described by the equation $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ is πab.

   (Note: This question is the ones studying math for the FUN!!!)